Section 3.11

Related Rates

The class of problems known as Related-Rate Problems consist of a desired and known rates of change (derivative) of two or more variables having an implicit relationship. If one strips away the word problems, it then becomes an exercise in implicit differentiation.

There is a general procedure for tackling related-rates problems.

Procedure 3.11.1. Follow these steps to solve most related-rate problems:

1. Find what you known and don't know.
   (a) Sketch a geometric picture to represent the problem.
   (b) Write down the rates that are given and the rate to be determined. Take note of the variables that differentiated $dy/dt, dx/dt$ etc. Use the units for context clues!
2. Write one or more equations relating the variable to be differentiated.
3. Form a relation between the rates by implicitly differentiating the equation in step 3.
4. Substitute the known values and known rates and solve for the desired rate.
5. Check the units and reasonableness of your solution.

Exercise 3.11.1. A 9 ft ladder leans against a wall. The bottom of the ladder is pulled away from the wall at a constant rate of 1 ft/s. At what rate is the top of the ladder moving (in the downwards direction) when the bottom is 5 ft from the wall? 9 ft from the wall? Desmos graph

Exercise 3.11.2. A spherical balloon is filled with helium at a constant rate of 100 in^3/min. What is the rate of change of its radius when the radius is 10 in?

Exercise 3.11.3. The balloon from Exercise 3.11.2 is released. An observer stands 200 meters from the launch site at an elevation equal to the elevation of the launch site. The balloon rises vertically at a constant rate of 4 m/s. How fast is the angle of elevation of the balloon increasing 30 seconds after launch?

Exercise 3.11.4. Blake wants to bake a cake in the shape of a frustum. He forms a cake pan by removing the top 4 inches from a hollow square pyramid with base width of 12 in and height of 8 in. He pours the batter at a constant rate of 15 in^3/sec. How fast is the level of batter rising the moment the pan is completely filled?

Exercise 3.11.5. An observer stands 20 m from the bottom of a 10 m tall Ferris wheel on a line perpendicular to the face of the Ferris wheel. The wheel revolves at a rate of .5 rev/min and the observer's line of sight with a specific seat on the wheel makes an angle θ with the ground. Forty seconds after that seat leaves the lowest point on the wheel, what is the rate of change of θ? Assume the observer's eyes are level with the bottom of the wheel.

Exercise 3.11.6. A piston is connected to a crank of radius 3 in via a 7 in connecting rod. The crank makes an angle θ with the line of motion of the piston. If the crank spins counterclockwise at a constant rate of 200 rev/sec, what is the speed of the piston when $\theta = \frac{\pi}{3}$?